CALCULUS I

Practice Problems
Integrals

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Preface

Here are a set of practice problems for my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

1. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,

2. Go to the download page for the site http://tutorial.math.lamar.edu/download.aspx and select the section you'd like solutions for and a link will be provided there.

3. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Integrals

Introduction

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Here is a list of topics in this chapter that have practice problems written for them.

Indefinite Integrals
Computing Indefinite Integrals
Substitution Rule for Indefinite Integrals
More Substitution Rule
Area Problem
Definition of the Definite Integral
Computing Definite Integrals
Substitution Rule for Definite Integrals

Indefinite Integrals

1. Evaluate each of the following indefinite integrals.

   (a) $\int 6x^5 - 18x^2 + 7\,dx$

   (b) $\int 6x^5\,dx - 18x^2 + 7$

2. Evaluate each of the following indefinite integrals.

   (a) $\int 40x^3 + 12x^2 - 9x + 14\,dx$

(b) $\int 40x^3 + 12x^2 - 9x\,dx + 14$

(c) $\int 40x^3 + 12x^2\,dx - 9x + 14$

For problems 3 – 5 evaluate the indefinite integral.

3. $\int 12t^7 - t^2 - t + 3\,dt$

4. $\int 10w^4 + 9w^3 + 7w\,dw$

5. $\int z^6 + 4z^4 - z^2\,dz$

6. Determine $f(x)$ given that $f'(x) = 6x^8 - 20x^4 + x^2 + 9$.

7. Determine $h(t)$ given that $h'(t) = t^4 - t^3 + t^2 + t - 1$.

Computing Indefinite Integrals

For problems 1 – 21 evaluate the given integral.

1. $\int 4x^6 - 2x^3 + 7x - 4\,dx$

2. $\int z^7 - 48z^{11} - 5z^{16}\,dz$

3. $\int 10t^{-3} + 12t^{-9} + 4t^3\,dt$

4. $\int w^{-2} + 10w^{-5} - 8\,dw$

5. $\int 12\,dy$

6. $\int \sqrt[3]{w} + 10\sqrt[5]{w^3}\,dw$

7. $\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\,dx$

8. $\int \frac{4}{x^2}+2-\frac{1}{8x^3}\,dx$

9. $\int \frac{7}{3y^6}+\frac{1}{y^{10}}-\frac{2}{\sqrt[3]{y^4}}\,dy$

10. $\int \left(t^2-1\right)\left(4+3t\right)dt$

11. $\int \sqrt{z}\left(z^2-\frac{1}{4z}\right)dz$

12. $\int \frac{z^8-6z^5+4z^3-2}{z^4}\,dz$

13. $\int \frac{x^4-\sqrt[3]{x}}{6\sqrt{x}}\,dx$

14. $\int \sin\left(x\right)+10\csc^2\left(x\right)dx$

15. $\int 2\cos\left(w\right)-\sec\left(w\right)\tan\left(w\right)dw$

16. $\int 12+\csc\left(\theta\right)\left[\sin\left(\theta\right)+\csc\left(\theta\right)\right]d\theta$

17. $\int 4\mathbf{e}^z+15-\frac{1}{6z}\,dz$

18. $\int t^3-\frac{\mathbf{e}^{-t}-4}{\mathbf{e}^{-t}}\,dt$

19. $\int \frac{6}{w^3}-\frac{2}{w}\,dw$

20. $\int \frac{1}{1+x^2}+\frac{12}{\sqrt{1-x^2}}\,dx$

21. $\int 6\cos\left(z\right)+\frac{4}{\sqrt{1-z^2}}\,dz$

22. Determine $f(x)$ given that $f'(x) = 12x^2 - 4x$ and $f(-3) = 17$.

23. Determine $g(z)$ given that $g'(z) = 3z^3 + \dfrac{7}{2\sqrt{z}} - \mathbf{e}^z$ and $g(1) = 15 - \mathbf{e}$.

24. Determine $h(t)$ given that $h''(t) = 24t^2 - 48t + 2$, $h(1) = -9$ and $h(-2) = -4$.

Substitution Rule for Indefinite Integrals

For problems 1 – 16 evaluate the given integral.

1. $\int (8x-12)\left(4x^2-12x\right)^4 dx$

2. $\int 3t^{-4}\left(2+4t^{-3}\right)^{-7} dt$

3. $\int (3-4w)\left(4w^2-6w+7\right)^{10} dw$

4. $\int 5(z-4)\sqrt[3]{z^2-8z}\, dz$

5. $\int 90x^2 \sin\left(2+6x^3\right) dx$

6. $\int \sec(1-z)\tan(1-z)\, dz$

7. $\int \left(15t^{-2}-5t\right)\cos\left(6t^{-1}+t^2\right) dt$

8. $\int \left(7y-2y^3\right)\mathbf{e}^{y^4-7y^2} dy$

9. $\int \dfrac{4w+3}{4w^2+6w-1} dw$

10. $\int \left(\cos(3t)-t^2\right)\left(\sin(3t)-t^3\right)^5 dt$

11. $\int 4\left(\frac{1}{z}-\mathbf{e}^{-z}\right)\cos\left(\mathbf{e}^{-z}+\ln z\right)dz$

12. $\int \sec^2(v)\,\mathbf{e}^{1+\tan(v)}\,dv$

13. $\int 10\sin(2x)\cos(2x)\sqrt{\cos^2(2x)-5}\,dx$

14. $\int \frac{\csc(x)\cot(x)}{2-\csc(x)}dx$

15. $\int \frac{6}{7+y^2}dy$

16. $\int \frac{1}{\sqrt{4-9w^2}}dw$

17. Evaluate each of the following integrals.

    (a) $\int \frac{3x}{1+9x^2}dx$

    (b) $\int \frac{3x}{\left(1+9x^2\right)^4}dx$

    (c) $\int \frac{3}{1+9x^2}dx$

More Substitution Rule

Evaluate each of the following integrals.

1. $\int 4\sqrt{5+9t}+12\left(5+9t\right)^7\,dt$

2. $\int 7x^3\cos\left(2+x^4\right)-8x^3\mathbf{e}^{2+x^4}\,dx$

3. $\displaystyle\int \frac{6\mathbf{e}^{7w}}{\left(1-8\mathbf{e}^{7w}\right)^3}+\frac{14\mathbf{e}^{7w}}{1-8\mathbf{e}^{7w}}\,dw$

4. $\displaystyle\int x^4-7x^5\cos\left(2x^6+3\right)dx$

5. $\displaystyle\int \mathbf{e}^z+\frac{4\sin\left(8z\right)}{1+9\cos\left(8z\right)}\,dz$

6. $\displaystyle\int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}}+7w^3-6\sqrt[3]{w}\,dw$

7. $\displaystyle\int \left(4+7t\right)^3-9t\sqrt[4]{5t^2+3}\,dt$

8. $\displaystyle\int \frac{6x-x^2}{x^3-9x^2+8}-\csc^2\left(\frac{3x}{2}\right)dx$

9. $\displaystyle\int 7\left(3y+2\right)\left(4y+3y^2\right)^3+\sin\left(3+8y\right)dy$

10. $\displaystyle\int \sec^2\left(2t\right)\left[9+7\tan\left(2t\right)-\tan^2\left(2t\right)\right]dt$

11. $\displaystyle\int \frac{8-w}{4w^2+9}\,dw$

12. $\displaystyle\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx$

13. $\displaystyle\int z^7\left(8+3z^4\right)^8\,dz$

Area Problem

For problems 1 – 3 estimate the area of the region between the function and the *x*-axis on the given interval using $n=6$ and using,

(a) the right end points of the subintervals for the height of the rectangles,
(b) the left end points of the subintervals for the height of the rectangles and,
(c) the midpoints of the subintervals for the height of the rectangles.

1. $f(x) = x^3 - 2x^2 + 4$ on $[1,4]$

2. $g(x) = 4 - \sqrt{x^2 + 2}$ on $[-1,3]$

3. $h(x) = -x\cos\left(\frac{x}{3}\right)$ on $[0,3]$

4. Estimate the net area between $f(x) = 8x^2 - x^5 - 12$ and the x-axis on $[-2,2]$ using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the x-axis?

The Definition of the Definite Integral

For problems 1 & 2 use the definition of the definite integral to evaluate the integral. Use the right end point of each interval for x_i^*.

1. $\int_1^4 2x + 3\,dx$

2. $\int_0^1 6x(x-1)\,dx$

3. Evaluate : $\int_4^4 \frac{\cos\left(\mathbf{e}^{3x} + x^2\right)}{x^4 + 1}\,dx$

For problems 4 & 5 determine the value of the given integral given that $\int_6^{11} f(x)\,dx = -7$ and $\int_6^{11} g(x)\,dx = 24$.

4. $\int_{11}^6 9f(x)\,dx$

5. $\int_6^{11} 6g(x) - 10f(x)\,dx$

6. Determine the value of $\int_2^9 f(x)\,dx$ given that $\int_5^2 f(x)\,dx = 3$ and $\int_5^9 f(x)\,dx = 8$.

7. Determine the value of $\int_{-4}^{20} f(x)\,dx$ given that $\int_{-4}^{0} f(x)\,dx = -2$, $\int_{31}^{0} f(x)\,dx = 19$ and $\int_{20}^{31} f(x)\,dx = -21$.

For problems 8 & 9 sketch the graph of the integrand and use the area interpretation of the definite integral to determine the value of the integral.

8. $\int_{1}^{4} 3x - 2\,dx$

9. $\int_{0}^{5} -4x\,dx$

For problems 10 – 12 differentiate each of the following integrals with respect to *x*.

10. $\int_{4}^{x} 9\cos^2\left(t^2 - 6t + 1\right)dt$

11. $\int_{7}^{\sin(6x)} \sqrt{t^2 + 4}\,dt$

12. $\int_{3x^2}^{-1} \frac{\mathbf{e}^t - 1}{t}\,dt$

Computing Definite Integrals

1. Evaluate each of the following integrals.

a. $\int \cos(x) - \frac{3}{x^5}\,dx$

b. $\int_{-3}^{4} \cos(x) - \frac{3}{x^5}\,dx$

c. $\int_{1}^{4} \cos(x) - \frac{3}{x^5}\,dx$

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

2. $\int_{1}^{6} 12x^3 - 9x^2 + 2\,dx$

3. $\int_{-2}^{1} 5z^2 - 7z + 3\,dz$

4. $\int_{3}^{0} 15w^4 - 13w^2 + w\,dw$

5. $\int_{1}^{4} \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\,dt$

6. $\int_{1}^{2} \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\,dz$

7. $\int_{-2}^{4} x^6 - x^4 + \frac{1}{x^2}\,dx$

8. $\int_{-4}^{-1} x^2\left(3 - 4x\right)dx$

9. $\int_{2}^{1} \frac{2y^3 - 6y^2}{y^2}\,dy$

10. $\int_{0}^{\frac{\pi}{2}} 7\sin\left(t\right) - 2\cos\left(t\right)dt$

11. $\int_{0}^{\pi} \sec\left(z\right)\tan\left(z\right) - 1\,dz$

12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2\left(w\right) - 8\csc\left(w\right)\cot\left(w\right)dw$

13. $\int_{0}^{2} \mathbf{e}^x + \frac{1}{x^2 + 1}\,dx$

14. $\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy$

15. $\int_{0}^{4} f\left(t\right)dt$ where $f\left(t\right) = \begin{cases} 2t & t > 1 \\ 1 - 3t^2 & t \le 1 \end{cases}$

16. $\int_{-6}^{1} g(z)\,dz$ where $g(z) = \begin{cases} 2-z & z > -2 \\ 4\mathbf{e}^{z} & z \le -2 \end{cases}$

17. $\int_{3}^{6} |2x-10|\,dx$

18. $\int_{-1}^{0} |4w+3|\,dw$

Substitution Rule for Definite Integrals

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

1. $\int_{0}^{1} 3\left(4x+x^4\right)\left(10x^2+x^5-2\right)^6\,dx$

2. $\int_{0}^{\frac{\pi}{4}} \frac{8\cos(2t)}{\sqrt{9-5\sin(2t)}}\,dt$

3. $\int_{\pi}^{0} \sin(z)\cos^3(z)\,dz$

4. $\int_{1}^{4} \sqrt{w}\,\mathbf{e}^{1-\sqrt{w^3}}\,dw$

5. $\int_{-4}^{-1} \sqrt[3]{5-2y} + \frac{7}{5-2y}\,dy$

6. $\int_{-1}^{2} x^3 + \mathbf{e}^{\frac{1}{4}x}\,dx$

7. $\int_{\pi}^{\frac{3\pi}{2}} 6\sin(2w) - 7\cos(w)\,dw$

8. $\int_{1}^{5} \frac{2x^3+x}{x^4+x^2+1} - \frac{x}{x^2-4}\,dx$

9. $\displaystyle\int_{-2}^{0} t\sqrt{3+t^2}+\frac{3}{\left(6t-1\right)^2}\,dt$

10. $\displaystyle\int_{-2}^{1}\left(2-z\right)^3+\sin\left(\pi z\right)\left[3+2\cos\left(\pi z\right)\right]^3\,dz$