Calc AB: Unit 5 Test Review!

Name _____

Period _____ Date _____

STAPLE THIS SHEET TO THE FRONT OF YOUR WORK.

On the back of this sheet is a detailed list of topics we have covered this unit.
To study, you should summarize and practice the material from the unit.
Use the guide below (and on the back of this sheet) to find and summarize major topics with notes, examples, comments, discussions of previous mistakes, etc.

Make sure your work is clear so that you'll be able to understand it when we use this again in April / May to study for the AP test.

Recommended length:	
½ of one side of one page	**1. Basic Integration**
½ of one side of one page	**2. Applications of Integrals**
½ of one side of one page	**3. First & Second Fundamental Theorems of Calculus**
(a more detailed list of topics can be found on the back of this page)	
as much as needed :)	**4. Additional Practice Problems (Attached)**
Required total length:	**1½ PAGES OF NOTES + ALL PRACTICE PROBLEMS**

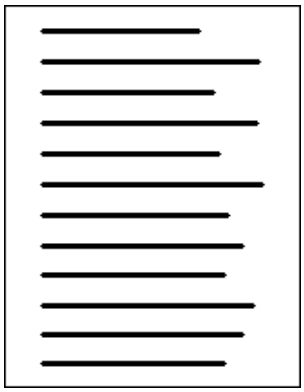
What to include:

Notes:
Summarize the most important concepts, vocabulary, etc., related to that topic.

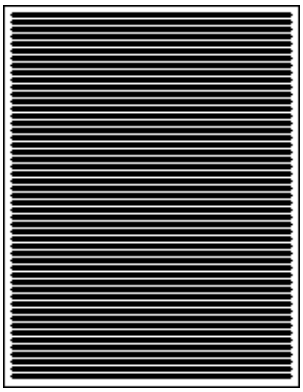
Worked-out examples:
You may use problems from homework, quizzes, the book, etc., or problems you make up yourself.

Discussions of previous mistakes:
Look at homework, classwork, and quizzes and find common mistakes you made.
Discuss the mistake to convince me (and yourself) that you won't make the same mistake again.

Etc.:
Things you often forget, or anything else you think might be helpful to remember (now *or* later in the year).



Not Enough
There's no way this covers it all.



Too Much
More is not better; it's just overwhelming!

Juuuuuust Right
Goldilocks would love this paper.

1. **Basic Integration**
   - Phrasing of questions
     - $\int f(x)dx$ vs. $\int_a^b f(x)dx$
     - Given $y' = blah$; find y
     - Solve the differential equation $\frac{dy}{dx} = blah$
     - Find the area under the curve / find the area of the region enclosed by *blah*
   - Geometry shortcuts
   - The 3 basic rules
     - sum / difference rule for integrals
     - constant multiple rule for integrals
     - power rule for integrals
   - *u*-substitution (4.5)
     - choosing a good *u*
     - process for *u*-substitution
     - when to rewrite a problem to be able to use the 3 basic rules instead of *u*-substitution
   - Using initial conditions (e.g., given $f(0) = 4$ or given a point on the graph of f)
   - Finding definite integrals on the graphing calculator

2. **Applications of Integrals**
   - Average value
   - Mean Value Theorem for Integrals
   - Given $\int_a^b f(x)dx$, find integrals of transformations of f (e.g., p. 292 #55-59)
   - Position / Velocity / Acceleration (a.k.a. rectilinear motion)
     - net distance vs. total distance and applications to word problems
     - average velocity / acceleration vs. instantaneous velocity / acceleration
     - accurate graphs (e.g., position based on velocity or velocity based on acceleration) (more accurate than the "basic shape" graphs from first semester)
     - *review practice AP free response questions and PVA stuff you learned first semester*

3. **First and Second Fundamental Theorems of Calculus**
   - First FTC
     - uses
     - various ways of writing/applying it (we saw three ways to write it)
   - Second FTC
     - derivation of Second FTC from First FTC
     - the definite integral as a function
       - notation
       - graphing $\int_a^x f(t)dt$ given graph of f
         (again, more accurate than the "basic shape" graphs from first semester)
       - analyzing the graph of the definite integral (e.g., p. 293 #73-74)
     - shortcuts for derivative of an integral that goes…
       - …from a constant to x ← simple
       - …from a constant to $u(x)$ ← more general
       - …from $v(x)$ to $u(x)$ ← most general
     - verifying the second FTC (e.g., p. 293 #75-80)

How should I study?

Do *lots* of textbook problems to practice your Algebra *and* Calculus skills.
Review notes, homework, quiz, and practice AP problems.
Identify your common mistakes and how to avoid them in the future.
Review how we have justified answers in class. (What is necessary? What isn't?)

Calc AB: Unit 5 Test Review! Additional Practice Problems * Required! *

Do this review on *separate paper. (You can use both sides of your paper.)*

Work done on <u>this</u> sheet will NOT be graded!

Problems must be easy for me to find, so **<u>USE THE TEMPLATE BELOW</u>**.

If you do a problem out of order, go to where it *should be* and put a note telling me where it *is*.

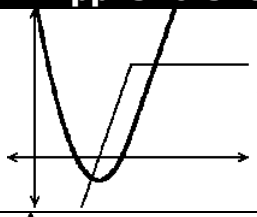
	Calculus AB: Unit 5: Test Review	Name
		Date
page #		
problem #	Show work clearly (you *don't* have to copy the problem)	
		BOX your answer
problem #		

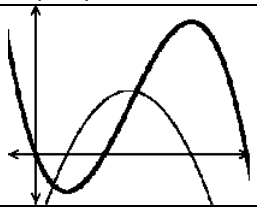
pp. 316-318	basic indefinite integrals (no *u*-substitution)	#1-9 all
	position / velocity / acceleration	#13, 14
	geometry shortcuts	#39, 40
	integrals of transformations	#41, 42
	basic definite integrals (no *u*-substitution)	#44-60 multiples of 4
	average value / MVT for integrals	#61, 62
	Second Fundamental Theorem of Calculus	#63-66 all* ↓
	* ALSO find $F(x)$ and differentiate it in order to *verify* 2nd FTC	
	more indefinite & definite integrals	#68-88 even
p. 293	analyzing the graph of $\int_a^x f(t)dt$	#73, 93, 94
	Second FTC + chain rule	#87, 89, 91

Answers are on the back of this page

KEY – TEXTBOOK PROBLEMS

pp. 316-318

1.  one part quadratic, one part linear

2. 

3. $\frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$

4. $\left(\frac{3}{\sqrt[3]{3}}\right)x^{2/3} + C$
or $3^{2/3}x^{2/3} + C$

5. $\frac{1}{2}x^2 - \frac{1}{x} + C$

6. $\frac{1}{2}x^2 - 2x - \frac{1}{x} + C$

7. $2x^2 + 3\cos x + C$

8. $5\sin x - 2\tan x + C$

9. $-x^2 + 2$

13. $240\ ^{\text{ft}}/_{\text{sec}}$

14. 211.2 feet

39. triangle; $\frac{25}{2}$

40. half-circle; 8π

41. **a.** 13
b. 7
c. 11
d. 50

42. **a.** 3
b. 1
c. 0
d. 10

44. $\frac{14}{3}$

48. $\frac{1}{8}$

52. 10 (geometry or calculus)

56. $\frac{4}{15}$ (calculus)

pp. 316-318, continued

60. $\sqrt{3}$

61. average value is $\frac{2}{5}$
$x = \frac{25}{4}$

62. average value is 2
$x = \sqrt[3]{2}$

63. $F(x) = \frac{2}{9}(1+x^3)^{3/2} - \frac{2}{9}$
$F'(x) = x^2(1+x^3)^{1/2}$ ✓

64. $F(x) = -\frac{1}{x} + 1$
$F'(x) = x^{-2}$ ✓

65. $F(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + \frac{3}{2}$
$F'(x) = x^2 + 3x + 2$ ✓

66. $F(x) = -\cot x - \text{undefined}$
$F'(x) = \csc^2 x$

68. $\frac{1}{3}x^3 + 2x - \frac{1}{x} + C$
(no *u*-substitution needed)

70. $\frac{2}{9}(x^3+3)^{3/2} + C$

72. $\frac{-1}{2(x^2+6x-5)} + C$

74. $-\frac{1}{6}\cos(3x^2) + C$

76. $2\sqrt{\sin x} + C$

78. $\frac{1}{2}\sec 2x + C$

80. $-\frac{1}{5}\cot^5 \alpha + C$

82. $\frac{5}{4}$

84. $\frac{2\sqrt{7}-1}{3}$

86. $\frac{32\pi}{105}$

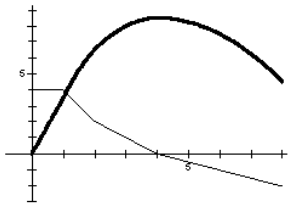
88. 0

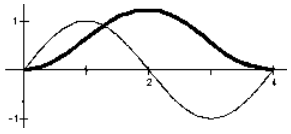
p. 293

73. **a.** 0; 7; 9; 8; 5

b. inc. on (0,4)
dec. on (4,8)

c. relative max:
$g(4) = 9$

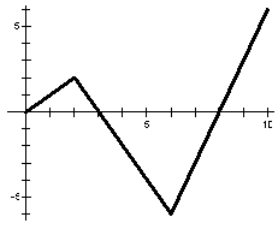
d. 

93. 
rel. max @ $x = 2$

94. **a.**

x	1	2	3	4	5
$g(x)$	1	2	0	-2	-4

x	6	7	8	9	10
$g(x)$	-6	-3	0	3	6

b. 

c. abs min: $g(6) = -6$
g' changes + to –
& $g(6)$ is more than $g(0)$ and $g(10)$

d. abs max: $g(10) = 6$
relative max @ $x = 2$,
but $g(10) > g(2)$,
and $g(10) > g(0)$

e. (6,10)
because that is where $g'(x) = f(x)$ is greatest

f. $x = 0$, 3, and 8

87. 8

89. $\cos x\sqrt{\sin x}$

91. $3x^2 \sin x^6$